Spring of Differential Equations in Gdańsk

18-21 marca 2015 r.

Centrum Zastosowań Matematyki

Politechnika Gdańska, Wydział Fizyki Technicznej i Matematyki Stosowanej

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Spis prelegentów

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- Wacław Marzantowicz (Uniwersytet im. Adama Mickiewicza w Poznaniu)
- Ewa Schmeidel (Uniwersytet w Białymstoku)
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2 Wykłady gości zagranicznych

2.1 Integrable systems and canonic quantization

Julia Bernatska, National University of Kyiv-Mohyla Academy

A number of nonlinear partial differential equations serve as universal physical models, describing a great variety of phenomena. These are soliton-type equations: the Korteweg-de Vries equation, the sine-Gordon equation, the nonlinear Schrödinger equation, the continuum Heisenberg model described by the isotropic Landau-Lifshits equation, the Boussinesq equation, the Toda model etc. They are called completely integrable equations due to infinitely many integrals of motion. Every mentioned equation represents a hierarchy of integrable Hamiltonian systems, and can be constructed on a coadjoint orbit of a loop group by the orbit method. This method gives a powerful apparatus for obtaining periodic and multisoliton solutions for such equations and solving other important problems - one of them is the problem of quantization on a Lagrangian manifold [1], that is a canonical quantization.

The controversial question how to choose a proper Lagrangian manifold for canonical quantization is evidently solved in terms of variables of separation (Darboux coordinates). For integrable Hamiltonian systems constructed by means of the orbit method we have a definite procedure for obtaining variables of separation, they are points of the spectral curve connected to a system. A half of them parametrizes the Liouville torus of the integrable system, thus the Liouville torus serves as a Lagrangian manifold. And the complexified Lagrangian manifold is a generalized Jacobian of the spectral curve, which coincides with the phase space of the system.

The canonical quantization of an integrable system gives a representation of its phase space symmetry algebra over the space of functions on a Lagrangian manifold. Using the Liouville torus as a Lagrangian manifold guarantees that the representation space consists of holomorphic functions - they are defined on the generalized Jacobian serving as the phase space of the system. The obtained representation is indecomposable and non-exponentiated [2].

As an example we consider the nonlinear Schrödinger equation and the continuum Heisenberg model, connecting to the same spectral curve, that allows to use the same Lagrangian manifold for canonical quantization. We construct the corresponding phase space symmetry algebras and their representations over the space of holomorphic functions on the Lagrangian manifold. Harmonic analysis on the representation space leads to

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the Whittaker equation (in some particular case) and its generalization with an irregular singularity.

Literatura

[1] M. Karasev, *Quantization by membranes and integral representations of wave functions, Quantization and infinite-dimensional systems*. Proceedings of the 12th summer workshop on differential geometric methods in physics, Bialowieza, Poland, 1993, 9–19.

[2] J. Bernatska, P. Holod, *Harmonic analysis on Lagrangian manifolds of Integrable Hamiltonian systems*, Journal of Geometry and Symmetry in Physics, 2013, Vol. 29, 39–51 (arXiv:1307.1785 [math-ph]).

2.2 How to construct stochastic models? Theory and numerics

Jacky Cresson, Université of Pau and Pays de l'Adour

General background: modelling of stochastic perturbations and validation of models

The aim of this lecture is to present a set of mathematical tools and results permitting to construct a stochastic viable model from a known deterministic one and second to validate them via numerical simulations. Indeed, many dynamical systems in Biology or Physics are in a first approach modelled by a deterministic dynamical systems. We can cite for example the classical Hodgkin-Huxley model in Biology or the Landau-Lipshitz model in Physics. Then, a set of observations are made and lead to the introduction of a stochastic or random component. For the Hodgkin-Huxley model this comes from the fact that there exists an intrinsic stochastic bioelectrical activity of neurons which is observed experimentally. For the Landau-Lifshitz model this comes from the fact that the electromagnetic fields behaves very randomly. However, to take into account these stochastic effects is in general not an easy task.

The Lecture is made of two parts : the first one deals with the construction of the stochastic model. The second one is devoted to numerical methods designed in order to validate these models. All the mathematical tools and results will be illustrated by numerous examples coming from Biology, Physics, Astronomy and Celestial Mechanics.

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Part I - Admissible or viable stochastic models

In a first part of this Lecture, we discuss such a modelling in the context of the theory of stochastic differential equations.

Returning to the initial deterministic model is always interesting. Indeed, it provides a set of constraints which are in general considered as fundamental by the scientists. This can be some fundamental law of Physics like conservation of energy, existence of some symmetries or invariance properties. At least these properties are in general respected by the deterministic model and a natural way to extend such a model in the stochastic framework is to construct a suitable stochastic perturbation respecting such a constraints in an appropriate sense.

In this Lecture, we will provide many results characterizing stochastic perturbations which preserve important properties : invariance of domain (in particular positivity), first integrals, variational structures (Lagrangian or Hamiltonian), symmetries, etc. These results will then be applied in various fields : Biology (behaviour of neurons, HIV population dynamics, Virus transmission models and models for the immune system, Cellular signaling networks, Population growth models, Tumor growth), Physics (Ferromagnetism), Astronomy and Celestial Mechanics (Two-body problem, Orbits of Satellites, Earth's rotation).

Part II - Numerical methods and validation of models

In a second part of this Lecture, we discuss how to make numerical simulations in order to validate these stochastic models. The main difficulty which is not usually discussed in the literature, is to provide numerical methods respecting the constraints of the models. This problem is well known in Hamiltonian mechanics where the conservation of energy is an important feature of the models and has leaded to the theory of variational integrators. For invariance of domains, symmetries, etc, the state of the arts is not so clear even in the deterministic case.

In this Lecture, we will discuss the construction of variational integrators in the context of the theory of discrete embeddings ans secondly we will discuss the construction of topological numerical scheme which are reminiscent of a general program initiated by R. Mickens around non-standard numerical scheme. These methodes will be discussed in the deterministic and stochastic case with numerous examples.

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2.3 Structured models for mathematical epidemiology

József Z. Farkas, University of Stirling

In recent years we have been working on the formulation an analysis of structured population models for infectious disease dynamics. In contrast to previous models, where for example the age of infection have been used as a structuring variable, we introduce structuring of the population with respect to infection (bacterium/virus) load and/or infectiousness.

In this talk we will focus on the models. First we will introduce the so called Wentzell (or Feller) boundary conditions in a structured population model with diffusion. The diffusion in our equation allows us to model random noise in a deterministic fashion. The power of Wentzell boundary condition is that it allows to incorporate a boundary state, which carries mass, namely the population of the uninfected individuals, in an elegant fashion. First we will consider a general linear model, then we will consider a nonlinear model which arises when modelling Wolbachia infection dynamics. We establish existence of solutions and consider the existence of positive steady states of the model. if time permits we will briefly mention a general framework we developed recently to treat models with 2-dimensional but non-monotone nonlinearities.

In the second part we will introduce and investigate an SIS-type model for the spread of an infectious disease, where the infected population is structured with respect to the different strain of the virus/bacteria they are carrying. Our aim is to capture the interesting scenario when individuals infected with different strains cause secondary (new) infections at different rates. Therefore, we consider a nonlinear infection process, which generalizes the bilinear process arising from the classic mass-action assumption. Our main motivation is to study competition between different strains of a virus/bacteria. From the mathematical point of view, we are interested whether the nonlinear infection process leads to a well-posed model. We use a semilinear formulation to show global existence and positivity of solutions up to a critical value of the exponent in the nonlinearity. Furthermore, we establish the existence of the endemic steady state for particular classes of nonlinearities.

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2.4 Periodic oscillations of a forced pendulum: from existence to stability

Rafael Ortega, Universidad de Granada

Consider the differential equation

$$x'' + \beta \sin x = f(t)$$

where β is a positive parameter and $f(t)$ is a 2π-periodic function. This is a simple model frequently employed to illustrate the methods of Nonlinear Analysis. Results on the existence of periodic solutions are usually obtained by a combination of tools coming from Topology and Calculus of Variations. The goal of this talk is to show that these tools are also useful in the study of the stability of periodic solutions. Stability is understood in the Lyapunov sense. We will assume that the parameter satisfies $\beta \leq \frac{1}{4}$ and the function $f(t)$ has zero average. The main result says that there exists a stable 2π-periodic solution for almost every periodic function $f(t)$ with zero average. The phrase "for almost every periodic function" is understood in the sense of prevalence. For this reason the notion of set of zero measure in a Banach space of infinite dimensions will play a role. The condition $\beta < \frac{1}{4}$ is sharp. The conclusion of the theorem is not valid "for all periodic functions".

3 Wykładowcy krajowi

3.1 General model of a cascade of reactions with time delays: global stability analysis

Marek Bodnar, Uniwersytet Warszawski

The considered problem consists of a cascade of reactions with discrete as well as distributed delays, which arose in the context of Hes1 gene expression. For the abstract general model sufficient conditions for global stability are presented. The method is based on comparison of the behaviour of the system of delay differential equations with an appropriate discrete system. Then the abstract result is applied to the Hes1 model and the condition for global stability of the steady state is given.

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3.2 Relativity of arithmetics as a fundamental symmetry of physics

Marek Czachor, Politechnika Gdańska

Arithmetic operations can be defined in various ways, even if one assumes commutativity and associativity of addition and multiplication, and distributivity of multiplication with respect to addition. In consequence, whenever one encounters 'plus' or 'times' one has certain freedom of interpreting this operation. This leads to some freedom in definitions of derivatives, integrals and, thus, practically all equations occurring in natural sciences. A change of realization of arithmetics, without altering the remaining structures of a given equation, plays the same role as a symmetry transformation. An appropriate construction of arithmetics turns out to be particularly important for dynamical systems in fractal space-times. Simple examples from classical and quantum, relativistic and nonrelativistic physics are discussed.

3.3 Analysis of cancer-immune system interactions model

Urszula Foryś, Uniwersytet Warszawski

We consider simplified model of cancer-immune system interactions for non-immunogenic tumours. The model is described as a system of three ODEs. We study asymptotic behaviour of this system, including existence of steady states, local stability of these states and possibility of bifurcations. We present some results concerning global stability for the states reflecting healthy organism as well as presence of tumour cells in the organism.

3.4 On mathematical model of Bats' Roost Searching Strategies

Ewa Girejko, Politechnika Białostocka

On the basis of cavity roosting bats' behavior living in Bialowiea Forest a mathematical model of their searching strategies is presented. We present a dynamical system describing a way of roost finding, appropriate for a certain species of bats, which consists of two nonlinear difference recursive equations of a special form. In the paper, stability of stationary solutions of the considered system is examined. Stationary solutions in

a biological interpretation mean points, where are tree cavities with habitat conditions suitable for a certain species of bats. Attractors are tree cavities that are settled by animals, while repulsers are cavities without settlement. Presented results are illustrated by computer simulations.
Co-authors: Robert Jankowski and Ewa Schmeidel

3.5 The analysis of demand and inventory steering model and its application

Piotr Hachuła, Centrum Wiedzy Logistycznej, Instytut Logistyki i Magazynowania w Poznaniu

The demand and inventory steering model introduced by Ma-Feng has been analysed in regard to its application in real business case. It has been shown that the model can describe specific situation of a product with a time-limited sale and its sale stimulation. The case is modelled by a discrete dynamical system – three first order recurence equations showing dependence between changes of stock, demand and deliveries over time. Stability analysis conducted with numerical methods and biffurcation diagram is shown.

3.6 Optimal investment problem in multidimensional capital accumulation model

Marta Kornafel, Uniwersytet Ekonomiczny w Krakowie

We will present the solution of the optimal investment problem in multidimensional model of capital accumulation. The ageing process of capital may be irregular function, so we rely our analysis on viscosity approach. The stability result for viscosity solutions will be presented.

3.7 Modelling subdiffusion: from difference equations to fractional differential equations

Tadeusz Kosztołowicz, UJK w Kielcach

A discrete model of random walk appears to be a useful tool in modelling subdiffusion or normal diffusion. We base the model of subdiffusion on a random walk model in

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a system with both discrete time and space variables. The particle's random walk is then described by a set of difference equations which can be solved by means of the generating function method. Using the generating function obtained for these equations we pass from discrete to continuous time and space variables by means of the procedure presented in this contribution. Finally we get the subdiffusion differential equation with fractional time derivative.

3.8 A numerical approach to two sex age-structured populations models in a space of Radon measures

Karolina Kropielnicka, Uniwersytet Gdański

The two sex age-structured populations model in a space of Radon measures is roughly speaking a system of transport equations with nonlocal boundary conditions considered in a certain space of measures equipped with flat metric. A numerical method described in this talk is based on a splitting technique, where we deal with two semigroups. One is defined by the transport operator while the second one is obtained from the nonlocal boundary condition. This setting allows us to approximate the solution of the underlaying problem as a sum of Dirac delta functions. These are results of a joint work with Piotr Gwiazda from University of Warsaw.

3.9 The method of dynamic projection operators in the theory of hyperbolic systems of partial differential equations with variable coefficients

Sergey Leble, Politechnika Gdańska

We consider a generalization of the projecting operators method for the case of Cauchy problem for systems of 1D evolution differential equations of first order with variable coefficients. It is supposed that the coefficients dependence on the only variable x is weak, that is described by a small parameter introduction. Such problem corresponds, for example, to the case of wave propagation in a weakly inhomogeneous medium. As an example, we specify the problem to adiabatic acoustics. For the Cauchy problem, to fix unidirectional modes, the projection operators are constructed. The method of successive approximations (perturbation theory) is developed and based on pseudodifferential operators theory. The application of these projection operators allows to obtain approximate evolution equations corresponding to the separated directed waves.

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3.10 Mathematical modeling of calcium induced calcium influx waves in cells and tissues

Zbigniew Peradzyński, WAT Warszawa

We propose a mathematical theory of fast calcium waves of CICI type. According to the suggestion of L. F. Jaffe [1] , these waves are supported by the influx of calcium from the intercellular space by the stress activated ion channels located in the cell membrane. The local stretching of the membrane is evoked by a thin cross-linked actin network, the cortex, attached to the cell membrane. Myosin motors in this network are responsible for the appearance of contractile forces, depending on the calcium concentration. The thickness of the cortex is of the order of 100 nm, which is very small in comparison with the size of typical cells (10-20). Cells are also equipped with the systems of pumps pumping out the excess of calcium. The competition between these two processes and the diffusion lead to the appearance of the travelling waves. The model is based on a system of reaction diffusion system for calcium and buffer proteins coupled with the mechanical equations for the traction forces produced by the cortex. The important feature of t the system is the dynamic boundary condition which is responsible for the influx of calcium. It is interesting that the theory leads to homoclinic travelling waves (as observed in reality) without postulating additional equation for so called recovery variable as it is usually done in the theory of calcium induced calcium released waves (where the calcium is released from the internal stores located in the cell).

Literatura

[1] L.F. Jaffe, *Stretch-activated calcium channels relay fast calcium waves propagated by calcium-induced calcium influx*, Biol. Cell 99, 175-184 (2007)

3.11 Influence of the distributed time delays on the stability in the family of agniogenesis models

Monika J. Piotrowska, Uniwersyte Warszawski

We propose a family of angiogenesis models, that is a generalisation of Hahnfeldt *et al.* model. Considered family of models is a system of two differential equations with distributed time delays. The global existence and the uniqueness of the solutions are

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proved. Moreover, the stability of the unique positive steady state is examined in the case when delay distributions are Erlang or piecewise distributions. Theorems guaranteeing the existence of stability switches and occurrence of the Hopf bifurcation are proved. Theoretical results are illustrated by numerical analysis performed for the parameters estimated by Hahnfeldt *et al.* (*Cancer Res.*, 1999).

3.12 Quaternionic quantum wave and the Klein-Gordon equations in Cauchy-Navier elastic solid

Lucjan Sapa, AGH Kraków

We show that a quaternionic quantum fild theory can be rigorously derived from the classical balance equations in isotropic crystal, where the energy and momentum transport are described by the Cauchy-Navier equations. We find a mathematical quaternionic formula for coupling between compression and torsion of the diplacement. The formula allows for a spatially localized wave function that is equivalent to the particle. A quantum wave equation that generates the Klein-Gordon equation is derived. We show the self-consistent classical interpretation of wave phenomena.

3.13 Nonlocal BVPs

Robert Stańczy, Uniwersytet Wrocławski

We prove some existence and multiplicity results for a class of nonlocal BVPs involving the Dirichlet Laplace operator.

3.14 Multi-scale polymeric flows

Agnieszka Świerczewska-Gwiazda, Uniwersytet Warszawski

We will concentrate on a class of mathematical models for polymeric fluids, which involves the coupling of the Navier-Stokes equations for a viscous, incompressible, constant-density fluid with a parabolic-hyperbolic integro-differential equation describing the evolution of the polymer distribution function in the solvent, and a parabolic integro-differential equation for the evolution of the monomer density function in the solvent. The

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viscosity coefficient, appearing in the balance of linear momentum equation in the Navier–Stokes system, includes dependence on the shear-rate as well as on the weight-averaged polymer chain length. The system of partial differential equations under consideration captures the impact of polymerization and depolymerization effects on the viscosity of the fluid. We discuss the existence of global-in-time, large-data weak solutions under fairly general hypotheses.

3.15 The finite difference methods of computation of X-rays propagation through a system of many lenses

Paweł Wojda, Politechnika Gdańska

The propagation of X-ray waves through an optical system consisting of many X-ray refractive lenses and X-ray focusing is considered. For solving the problem of electromagnetic wave propagation, a finite-difference method for the paraxial wave equation is suggested and applied. The error of simulation is estimated mathematically and investigated. It is found out that very detailed difference mesh is necessary for reliable and accurate computation of propagation of X-ray waves through a system of many lenses. The reasons of necessity of very detailed difference mesh is that after the wave passes through a system of many lenses the electric field becomes a quickly oscillating function of coordinates perpendicular to the optical axis and very detailed difference mesh is necessary to digitize such a wave field. To avoid this diffculty, we introduce the equation for a complex phase function instead of the equation for an electric field. Equation for complex phase is nonlinear equation, in contrast to the paraxial wave equation. It is shown that equation for a phase function allows to considerably reduce the detail of difference mesh without loss in reliability and precision of simulations of X-rays propagation through the system of many lenses and the X-rays focusing. The simulation error of the suggested method is estimated and the examples of computation result are presented.

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4 Sesja plakatowa

4.1 Local versions of Banach principle and new methods of applications of contractive maps

Eugeniusz Barcz, UWM w Olsztynie

In the paper "Local versions of Banach principle and new methods of applications of contracive maps" in a shape of a poster we demonstrate generalizations of local versions of the Banach principle for nonlinear contractions adding new methods of applications of contractive maps.

Generalizations of local versions of Banach principle related to the so called γ-contractions are shown being the main results of this paper. The famous Cauchy problem is considered here for differential equation in Banach space. In the proof of this theorem we use the new method based on some local version of Banach principle (Theorem 2.5)

In this paper it also proved the inverse function theorem both in Banach spaces. The proofs presented here, based on the local versions of Banach principle, are essentially simpler.

4.2 Stability of Dirichlet problems involving fractional Laplacian

Dorota Bors, Uniwersytet Łódzki

We consider some Dirichlet problems for the differential equations with fractional Laplacian. Both existence and stability results are proved by the use of the varational methods.

4.3 Delayed differential equation with non-constant delay in biology

Antoni Leon Dawidowicz, Uniwersytet Jagielloński

Differential equations with delayed argument have numerous applications in biology, like e.g. in immunology, whether of epidemiology. These equation has the form

$$\frac{dx}{dt} = f(x(t), x(t - \tau))$$

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where the set X of values of function x may be olso multidimensional. We shall present the applications of more general equations i. e. equations of the form

$$\frac{dx}{dt} = F(x_t)$$

where F is defined on function space and $x_t : (-r, 0] \to X$ is defined by the formula

$$x_t(s) = x(t+s).$$

The examples of application of such equations will be presented. To such equations the classical method of steps cannot be used.

Co-authors: Anna Poskrobko (Uniwersytet w Białymstoku), Jerzy Leszek Zalasiński (Expert FAO)

4.4 Method of lines for Sobolev type equations

Danuta Jaruszewska-Walczak, Uniwersytet Gdański

We consider the first initial boundary value problem for the following nonlinear Sobolev type equation with a rapid growing nonlinearity

$$\frac{\partial}{\partial t}(\Delta u - u) + e^u = 0 \quad \text{on} \quad \Omega \times [0, T].$$

Here $\Omega \subset R^n$ is a bounded domain with smooth boundary and Δ is the Laplace operator with respect to the spatial variable. The unique solvability in the classical sense for this problem is proved by M. O. Korpusov and A. G. Sveshnikov. Estimates for the time of the blow-up are given.

We are interested in numerical solving such problem. Stability of the method of lines is investigated.

4.5 On the Cauchy problem for hyperbolic functional-differential equations

Adrian Karpowicz, Uniwersytet Gdański

We consider the Cauchy problem for a nonlocal wave equation in one dimension. We study the existence of solutions by means of bicharacteristics. The existence and uniqueness is obtained in $W^{1,\infty}_{loc}$ topology. The existence theorem is proved in a subset generated by certain continuity conditions for the derivatives.

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4.6 Applications of differential equations with fractional time derivative in describing subdiffusion processes

Katarzyna D. Lewandowska, Gdański Uniwersytet Medyczny

We show the applications of hyperbolic and parabolic subdiffusive equation with time fractional derivative to describe the transport process in membrane system and to study the subdiffusive impedance in electrochemical system. Based on solutions of the equations we find characteristic power functions which can be used to extract the subdiffusive parameters of the system from experimental data. To illustrate our considerations we find the values of subdiffusion parameters for a few media.
Co-author: Tadeusz Kosztołowicz

4.7 Bounded and stable solutions for nonlinear second order neutral difference equation

Magdalena Nockowska-Rosiak, Politechnika Łódzka

Using the techniques connected with the measure of noncompactness we investigate the neutral difference equation of the following form

$$\Delta\left(r_n\left(\Delta\left(x_n + p_n x_{n-k}\right)\right)^\gamma\right) + q_n x_n^\alpha + a_n f(x_{n+1}) = 0,$$

where $x : \mathbb{N}_k \to \mathbb{R}$, $a, p, q : \mathbb{N}_0 \to \mathbb{R}$, $r : \mathbb{N}_0 \to \mathbb{R} \setminus \{0\}$, $f : \mathbb{R} \to \mathbb{R}$ is continuous and k is a given positive integer, $\alpha \geq 1$ is a ratio of positive integers with odd denominator, and $\gamma \leq 1$ is ratio of odd positive integers; $\mathbb{N}_k := \{k, k+1, \dots\}$. Sufficient conditions for the existence of a bounded or stable of a special type solution are presented.
Co-authors: Marek Galewski, Robert Jankowski (Politechnika Łódzka) Robert Jankowski, Ewa Schmeidel (Uniwersytet w Białymstoku)

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4.8 Generalized upper and lower solutions for discontinuous ordinary differential equations

Krzysztof A. Topolski, Uniwersytet Gdański

First order discontinuous ordinary differential equations are considered. By considering noncontinuous sub and supersolutions (upper and lower absolutely continuous functions) we obtain more general results than in standard theory. We present theorems on the existence of extremal solutions for a large class of boundary value problems. We assume neither continuity nor monotonicity of boundary functions.

4.9 The APD-RMSD descriptor for molecules' and ligands' conformationn

Rafał D. Urniaż, Uniwersytet Medyczny w Lublinie

There exists a deep need for computational methods that accurately represent the spatial relationships between different biological structures, such as molecules and ligands. To cater this demand, the APD-RMSD descriptor was proposed. This approach incorporates two different types of descriptors: the angle-plane deviation (APD) and root-mean-square deviation (RMSD). Due to, the synergic description of angles (APD) and distance (RMSD) conformational changes of the molecules could be described more precisely. The method was validated and successfully applied to parameterize the model of the glucokinase (GCK) and its regulatory protein (GCKR). The analysis was implemented in dedicated computational environment called $\texttt{Grow}_4$ being a molecular modeling platform available free of charge at www.grow4.eu. The project was founded by Rafal Urniaz and subsequently it is developed at Medical University of Lublin (Poland).

4.10 Integro–differential inequalities related to the least positive eigenvalue of some eigenvalue problems

Damian Wiśniewski, UWM w Olsztynie

We consider the eigenvalue problem for the Laplace–Beltrami operator Δ_ω on the unit sphere Ω:

$$\begin{cases} \Delta_\omega \psi + \lambda(\lambda + n - 2)\psi(\omega) = 0, & \omega \in \Omega; \\ \frac{\partial \psi}{\partial \vec{\nu}} + (\lambda \chi(\omega) + \gamma(\omega))\psi(\omega) = 0, & \omega \in \partial\Omega \end{cases} \qquad (EVP_1)$$

and for $m(x)$-Laplacian Dirichlet problem:

$$\begin{cases} \mathrm{div}_\omega(|\nabla_\omega \psi|^{m(\omega)-2}\nabla_\omega \psi) + \vartheta|\psi|^{m(\omega)-2}\psi = 0, & \omega \in \Omega; \\ \psi(\omega) = 0, & \omega \in \partial\Omega. \end{cases} \tag{EVP_2}$$

We have proved the existence of the smallest positive eigenvalue of (EVP_1) and Friedrichs-Wirtinger-type inequalities corresponding to our problems. We have also derived some integro-differential inequalities related to the smallest positive eigenvalue of (EVP_1) and (EVP_2).
Co-author: Mariusz Bodzioch

4.11 Method of lines for hyperbolic stochastic functional partial differential equations

Monika Wrzosek, Uniwersytet Gdański

We consider the initial value problem for first-order stochastic functional partial differential equation driven by Brownian motion

$$\begin{aligned} \frac{\partial u}{\partial t}(t,x) + a(t,x)\frac{\partial u}{\partial x}(t,x) &= f(t,x,u_{(t,x)}) + g(t,u_{(t,0)})\dot{W}_t, \quad (t,x) \in [0,T] \times \mathbb{R} \\ u(t,x) &= \varphi(t,x), \quad (t,x) \in [-r,0] \times \mathbb{R}, \end{aligned}$$

where $\dot{W}_t$ is white noise and $u_{(t,x)}$ is a Hale-type operator

$$u_{(t,x)}(\tau,\theta) = u(t+\tau, x+\theta) \quad for \quad (\tau,\theta) \in [-r,0] \times \mathbb{R}.$$

We apply the method of lines and prove the stability of the numerical scheme. This result is proved with the help of representation, existence and uniqueness, and the estimation of solution lemmas.
Co-author: Maria Ziemlańska

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4.12 Nonoscillatory bounded solutions of k-dimensional system of neutral difference equations

Małgorzata Zdanowicz, Uniwersytet w Białymstoku

The k-dimensional system of neutral type nonlinear difference equations with delays in the following form

$$\begin{aligned} \Delta(x_i(n) + p_i(n)x_i(n-\tau_i)) &= a_i(n)f_i(x_{i+1}(n-\sigma_i)) + g_i(n), \quad i = 1, \dots, k-1 \\ \Delta(x_k(n) + p_k(n)x_k(n-\tau_k)) &= a_k(n)f_i(x_1(n-\sigma_k)) + g_k(n) \end{aligned}$$

is considered. The aim of this paper is to present sufficient conditions for the existence of nonoscillatory bounded solutions of the system above with various $[p_1(n), \dots, p_k(n)]$
Co-authors: Ewa Schmeidel, Małgorzata Zdanowicz

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4.13 Nonlocal Robin problem for weak quasilinear elliptic equations in a plane domain

Krzysztof Żyjewski, UWM w Olsztynie

Let $G \subset \mathbb{R}^2$ be a bounded domain. We assume that the boundary $\partial G = \overline{\Gamma}_+ \cup \overline{\Gamma}_-$ is a smooth curve everywhere except at the origin $\mathcal{O} \in \partial G$ and near the point $\mathcal{O}$ curves $\Gamma_\pm$ are lateral sides of an angle with the measure $\omega_0 \in [0, 2\pi)$ and the vertex at $\mathcal{O}$; near $\mathcal{O}$ the curve $\sigma_0 = G \cap \{x_2 = 0\}$.

We shall consider a weak quasilinear elliptic equation with the nonlocal boundary condition connecting the values of the unknown function u on the curves $\Gamma_\pm$ with its

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values of u on the σ_0.

$$\begin{cases} -\frac{d}{dx_i}(a^{ij}(x)|u|^q u_{x_j}) + a(x,u,\nabla u) = 0, \quad x \in G \\ \\ \frac{\partial u}{\partial \nu} + \frac{\beta_\pm}{|x|}u|u|^q + \frac{b_\pm}{|x|}u\big(\gamma_\pm(x)\big)\left|u\big(\gamma_\pm(x)\big)\right|^q = g_\pm(x,u), \quad x \in \Gamma_\pm; \end{cases} \tag{QL}$$

here:

- $q \geq 0,\ \beta_\pm > 0,\ b_\pm \geq 0;$
- $\gamma_\pm$ are diffeomorphisms mapping of $\Gamma_\pm$ onto σ_0.

We investigate the behavior of weak solutions of the above problem in a neighborhood of the boundary corner point $\mathcal{O}$.

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