Mathematical modeling of calcium induced calcium influx waves in cells and tissues

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We propose a mathematical theory of fast calcium waves of CICI type. According to the suggestion of L. F. Jaffe [1], these waves are supported by the influx of calcium from the intercellular space by the stress activated ion channels located in the cell membrane. The local stretching of the membrane is evoked by a thin cross-linked actin network, the cortex, attached to the cell membrane. Myosin motors in this network are responsible for the appearance of contractile forces, depending on the calcium concentration. The thickness of the cortex is of the order of 100 nm, which is very small in comparison with the size of typical cells (10-20 μ m). Cells are also equipped with the systems of pumps pumping out the excess of calcium. The competition between these two processes and the diffusion lead to the appearance of the travelling waves. The model is based on a system of reaction diffusion system for calcium and buffer proteins coupled with the mechanical equations for the traction forces produced by the cortex. The important feature of t the system is the dynamic boundary condition which is responsible for the influx of calcium. It is interesting that the theory leads to homoclinic travelling waves (as observed in reality) without postulating additional equation for so called recovery variable as it is usually done in the theory of calcium induced calcium released waves (where the calcium is released from the internal stores located in the cell).

[1] L.F. Jaffe, "Stretch-activated calcium channels relay fast calcium waves propagated by calciuminduced calcium influx", Biol. Cell 99, 175-184 (2007).