Integro - differential inequalities related to the least positive eigenvalue of some eigenvalue problems

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Abstract

We consider the eigenvalue problem for the Laplace-Beltrami operator Δ_ω on the unit sphere Ω:

$$(EVP_1) \qquad \begin{cases} \Delta_\omega \psi + \lambda(\lambda + n - 2)\psi(\omega) = 0, & \omega \in \Omega; \\ \\ \frac{\partial \psi}{\partial \vec{\nu}} + (\lambda\chi(\omega) + \gamma(\omega))\psi(\omega) = 0, & \omega \in \partial\Omega \end{cases}$$

and for $m(x)$- Laplacian Dirichlet problem:

$$(EVP_2) \qquad \begin{cases} \mathrm{div}_\omega(|\nabla_\omega \psi|^{m(\omega)-2}\nabla_\omega \psi) + \vartheta|\psi|^{m(\omega)-2}\psi = 0, & \omega \in \Omega; \\ \\ \psi(\omega) = 0, & \omega \in \partial\Omega. \end{cases}$$

We have proved the existence of the smallest positive eigenvalue of (EVP_1) and Friedrichs - Wirtinger - type inequalities corresponding to our problems. We have also derived some integro-differential inequalities related to the smallest positive eigenvalue of (EVP_1) and (EVP_2).