

# Nonlocal Robin problem for weak quasilinear elliptic equations in a plane domain

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## Abstract

Let  $G \subset \mathbb{R}^2$  be a bounded domain. We assume that the boundary  $\partial G = \bar{\Gamma}_+ \cup \bar{\Gamma}_-$  is a smooth curve everywhere except at the origin  $\mathcal{O} \in \partial G$  and near the point  $\mathcal{O}$  curves  $\Gamma_{\pm}$  are lateral sides of an angle with the measure  $\omega_0 \in [0, 2\pi)$  and the vertex at  $\mathcal{O}$ ; near  $\mathcal{O}$  the curve  $\sigma_0 = G \cap \{x_2 = 0\}$ .

We shall consider a weak quasilinear elliptic equation with the non-local boundary condition connecting the values of the unknown function  $u$  on the curves  $\Gamma_{\pm}$  with its values of  $u$  on the  $\sigma_0$ .

$$\begin{cases} -\frac{d}{dx_i}(a^{ij}(x)|u|^q u_{x_j}) + a(x, u, \nabla u) = 0, & x \in G \\ \frac{\partial u}{\partial \nu} + \frac{\beta_{\pm}}{|x|} u |u|^q + \frac{b_{\pm}}{|x|} u(\gamma_{\pm}(x)) |u(\gamma_{\pm}(x))|^q = g_{\pm}(x, u), & x \in \Gamma_{\pm}; \end{cases}$$

here:

- $q \geq 0$ ,  $\beta_{\pm} > 0$ ,  $b_{\pm} \geq 0$ ;
- $\gamma_{\pm}$  are diffeomorphisms mapping of  $\Gamma_{\pm}$  onto  $\sigma_0$ .

We investigate the behavior of weak solutions of the above problem in a neighborhood of the boundary corner point  $\mathcal{O}$ .

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