Nonlocal Robin problem for weak quasilinear elliptic equations in a plane domain

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Abstract

Let $G \subset \mathbb{R}^2$ be a bounded domain. We assume that the boundary $\partial G = \Gamma_+ \cup \Gamma_-$ is a smooth curve everywhere except at the origin $O \in \partial G$ and near the point $O$ curves $\Gamma_{\pm}$ are lateral sides of an angle with the measure $\omega_0 \in [0, 2\pi)$ and the vertex at $O$; near $O$ the curve $\sigma_0 = G \cap \{x_2 = 0\}$.

We shall consider a weak quasilinear elliptic equation with the non-local boundary condition connecting the values of the unknown function $u$ on the curves $\Gamma_{\pm}$ with its values of $u$ on the $\sigma_0$.

$$
\begin{cases}
- \frac{d}{dx_i} (a_{ij}(x)|u|^q u_{x_j}) + a(x, u, \nabla u) = 0, \\
\frac{\partial u}{\partial \nu} + \beta_{\pm} |u|^q + b_{\pm} \frac{|u(\gamma_{\pm}(x))|}{|x|} = g_{\pm}(x, u),
\end{cases}
$$

$x \in G$

\begin{align*}
\frac{\partial u}{\partial \nu} + \beta_{\pm} |u|^q + b_{\pm} \frac{|u(\gamma_{\pm}(x))|}{|x|} = g_{\pm}(x, u),
\end{align*}

$x \in \Gamma_{\pm}$;

here:

- $q \geq 0$, $\beta_{\pm} > 0$, $b_{\pm} \geq 0$;
- $\gamma_{\pm}$ are diffeomorphisms mapping of $\Gamma_{\pm}$ onto $\sigma_0$.

We investigate the behavior of weak solutions of the above problem in a neighborhood of the boundary corner point $O$.

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