MATHEMATICA APPLICANDA
Vol. 42(2) 2014, p. 179–192
doi: 10.14708/ma.v42i2.600

Cezary Stępień (Warsaw)
Małgorzata Prolejko (Olsztyn)

The use of positioners in creating modular models of horns for mammals from the *Bovidae* family[1]

Abstract This paper describes a method suitable for creating animated modular models of horns for mammals belonging to the *Bovidae* family. Our method uses time-dependent positioners—fragments of modules with their own coordinate systems. Positioners are used in two ways: for placing modules appropriately next to each other and for creating the lateral surfaces of modules. Thanks to this double usage of the positioners, a continuous surface is achieved, regardless of the complexity of the time-dependent parameters. Different connections between parameters of modules are considered, justified from the point of view of modeling horns. The method is illustrated with the example of creating a time-dependent model of a ram's horn.

2010 Mathematics Subject Classification: Primary 03D20; Secondary 65D18; 68U05.

Key words and phrases: computer graphics, animation, modeling, geometric transformation, modular model, positioner, horn.

1. Introduction In this paper, we consider *modular models* of horns. This means that the model can be decomposed into modules, i.e. repetitive, similar fragments. The natural process of the growth of horns is described in [14]. It draws attention to its cyclical nature, which allows us to distinguish modules. They appear with a shift in time by a predetermined value, e.g. a year. A new tissue is formed at the base of the horn, in a place that we call an *active area*. The horn tissue created at a particular moment does not change its shape. It is only pushed away from the active area by younger growths. The periodicity of the growth process means the repetition of certain changes in the active area done a specific frequency. This process causes the formation of a linear hierarchy of modules, wherein the oldest module is furthest from the active area and the whole horn is fixed to the base of the youngest module (Fig. 1).

Static models of horns (as well as beaks and shells) composed with modules are described in [12], where it is assumed that all modules satisfy the condition of geometric similarity i.e. $V_i = sV_{i-1}$, $(i = 1, 2, ...k)$, where V_i is a set

[1]This publication is co-financed by the European Union as part of the European Social Fund within the project Center for Applications of Mathematics.

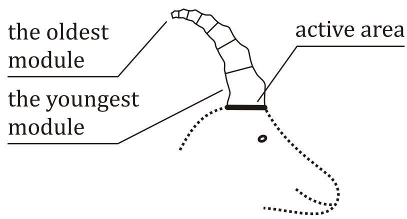

Figure 1: A linear hierarchical modular model of a horn.

describing the shape of module i and $s > 1$ is a scale factor. Each subsequent module is scaled relative to its predecessor by the same factor, greater than 1.

The Iterated Function System with Condensation method (IFSC) [1], [2] can be used to generate model satisfying the criteria. Under this method, a set that is the solution, is generated iteratively from the initial set. In each iteration, the set obtained in the previous iteration is processed by a finite number of given transformations. The result of the current iteration is a set which is a union of a specially predefined set (condensation set) and all the sets obtained by these transformations. IFSC, proposed in [12], gives satisfactory models of many natural formations. Its main advantage is the speed of creating models. In [13], a modification of this method is defined, which takes into account cases when the placement of one module relative to its predecessor depends on its index according to a predetermined function. This modification is useful in modeling such phenomena as deformation caused by the weight of previously formed parts [14]. Thus the created models are more akin the original outgrowths. Nevertheless, only static models are obtained, representing the appearance of the original outgrowth at a particular moment.

The method described in [12], [13], where geometrical similarity of the modules is assumed, often turns out to be too simplistic to make animated models of horns. In this paper, we create models that have a better fit to their prototypes.

The aim of this paper is to present the concept of the use of positioners, selected parts of the module with their own coordinate systems, to create animated models of horns. For this purpose, we focus on the fragment of a module which is in direct contact with the module previously formed. Due to the growth of the module, this fragment changes its position over time, which can be described by its matrix in the coordinate system of the module. By the concept of a positioner we mean both: the shape of this fragment and the description of its motion according to a time-dependent transformation matrix, called further the *motion matrix* for convenience.

An important feature of the presented method is the double usage of a single positioner. Firstly, we calculate the shape of a single module and secondly, we place the created structure appropriately in relation to the new module. Such an approach provides consistency of the compound solid at any time, as well as continuity of its lateral surface. It does not matter how complexly the parameters depend on time. The double usage of positioners simplifies the modeling process.

The concept of a positioner was proposed in [8], [9]. While maintaining the same method of indexation in the case of horns as in the case of non-branching plants, a positioner describes the position of the younger module in the coordinate system of the older one. Here we propose a different type of indexation, due to the fact that horns have different development rules than plants. Although in both cases each element appears after a certain predetermined delay in relation to the previous one, plants have the oldest module fixed (to the ground), horns have the youngest one fixed (to the head of an animal). In addition, in the case of plants, branching and non-branching models can be considered [10], [11].

The structure of this paper is as follows. In Section 2, we describe the growth process of a horn. In Section 3 we describe the notation used in the work. Next, in Section 4, we explain the concept of a positioner and the way it depends on time. A practical method of creating modules is described in Section 5. In Section 6 we explain how the hierarchical structure can be used to simplify the modeling process. Afterward, in Section 7, we show an example of creating a compound solid composed of modules, which is a model of a horn. This example confirms the appropriateness of our solutions. Finally, there is a discussion in Section 8.

2. The growth of horns In this paper, we deal with models of horns of animals from the *Bovidae* family. Horns are made of keratin, which is a product of the dermis. They are fixed to the skull. New layers of tissue are formed in the active area by the frontal sinus. This process moves the previously formed layers from the bone core. This type of horn does not branch.

We can observe movement of a previously formed fragment of the horn in relation to the active area, together with bending and twisting. This can be described by angles relative to the x, y and z axes of this system and by a translation vector. This is caused by the differences in the formation rate of tissue and by differences in the directions of growth in various parts of the active area [14]. Growth of the active area causes differences in the size of modules. The diversity of growth vectors in particular fragments of the active area causes rotation (Fig. 2).

Among the bovids, we can observe different shapes of horns. Regular ones can be described by a changing cross-section moved along a path, which is more or less a twisted logarithmic helix [14]. Others have paths resembling a circular helix (eg. mouflon) and still others have paths of curvature depending on the moment when a particular piece of a horn was created (e.g.

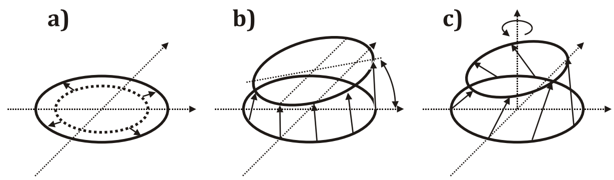

Figure 2: The impact of the speed and direction of growth on the shape of a module: a) growth of the active area, b) the cause of bending, c) the cause of twisting.

in goat horns, older fragments have a bigger curvature) [6]. In that paper, it was stated that the shape of a cross-section depends on a species and can vary from almost circular to almost triangular.

The fragment of a horn created at a particular moment reflects the shape of the active area at that moment, while the active area changes size over time. The rule is that the active area spreads with time, but there are exceptions. For example, the active area in a mouflon's horn grows for only approximately 6 years and sometimes then even decreases. The growth rate of the length of a horn, calculated along a path, depends on the age of the animal. The accretion in horns for goats and mouflons is the biggest between the first and the third year with the maximum in the second year. The growth rate is strictly related to the season. Ram's horns grow fastest in May and June, and almost stop between October and February. In winter, an annual ring is formed (e.g. in cows and rams) [14], [5]. Sometimes annual rings can be worn off [3]. Instead, we can often observe a different kind of ring, created more frequently.

3. The notation used in this paper In this paper we use matrix notation in a homogeneous coordinate system in order to describe 3D transformations such as translations and rotations [4]. In such a system the matrices have dimensions 4×4. The new coordinates of a point are calculated by multiplying its original coordinates $\mathbf{p} = (x, y, z, 1)^T$ by the appropriate motion matrix. We use translation and rotation matrices.

The translation matrix, indicating a shift of the vector (d_x, d_y, d_z) is denoted by the symbol $\mathbf{T}(d_x, d_y, d_z)$. Matrices describing rotation around the axes x, y, and z by the angles α, β and γ respectively are denoted by $\mathbf{R}_x(\alpha)$, $\mathbf{R}_y(\beta)$ and $\mathbf{R}_z(\gamma)$. These matrices have the following forms:

$$\mathbf{T}(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Further, we assume that the parameters d_x, d_y, d_z, α, β, γ can be time-dependent.

Let k denote the number of modules. We assume that k is predetermined. We denote modules (elements) by E_i ($i = 1, 2, \ldots\ k$). The shape of module E_i depends on time. It is described in its local coordinate system by the set $V_i(t)$. The position and the orientation of the module E_{i-1} in the coordinate system of the module E_i at a particular moment is determined by a certain transformation being the composition of both a translation and a rotation. We call this a *rigid body transformation.* As we suppose this transformation occurs in a time interval, we can name it a *motion transformation.*

4. The positioners We assume that in the process of creating a compound solid, modules appear sequentially one after another, each at a constant delay T relative to its predecessor. In the time interval $\big((i-1)T, iT\big]$ the compound solid consists of i modules, arranged in a linear hierarchy E_1, $E_2, \ldots\ E_i$. The shapes of the modules are described by time-dependent sets of points in 3D space. These sets are denoted by $V_1(t)$, $V_2(t), \ldots\ V_i(t)$.

By changing its shape, the newly created module E_i causes rotation and translation of the previous module E_{i-1} as well as the entire hierarchy.

The shape of the active area, where new tissue is formed, is described by the line $L(t)$. Further, we assume that it is continuous both in time and space.

Each module can be described in its local 3D coordinate system. Also, time dependencies are described on the local time-axis τ. We assume that each module develops in the interval $\tau \in [0, T]$ of its own local time. For $\tau < 0$ the corresponding module does not exist, and for $\tau > T$ it exists, but does not change its shape.

The module E_i appears at the moment $t = (i-1)T$, grows until moment T and then stops changing. For a predetermined time t and delay T, we have the following dependencies:

$$\left.\begin{aligned} i\colon\ (i-1)T < t \le iT \\ \tau_i = t - (i-1)T \end{aligned}\right\}. \tag{1}$$

Therefore, there is a linear correspondence between the global time axis and the local one. For module i: $\tau_i = 0 \Leftrightarrow t = (i-1)T$ and $\tau_i = T \Leftrightarrow t = iT$ (Fig. 3).

Let us assume, that the active area of the module E_i can be described in the interval $\tau \in [0, T]$ by a certain line $L_i(\tau)$. Consider the set of points of the module E_i in the coordinate system of the line $L_i(0)$. The set described in this way is denoted by $\bar{V}_i(\tau)$.

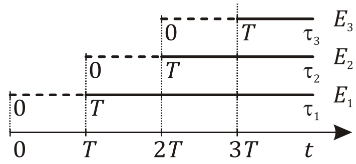

Figure 3: Local time axes τ_i of modules E_i against the global time axis t; the dashed line shows the stage of growth, and the solid line—the stage when the module has a constant shape.

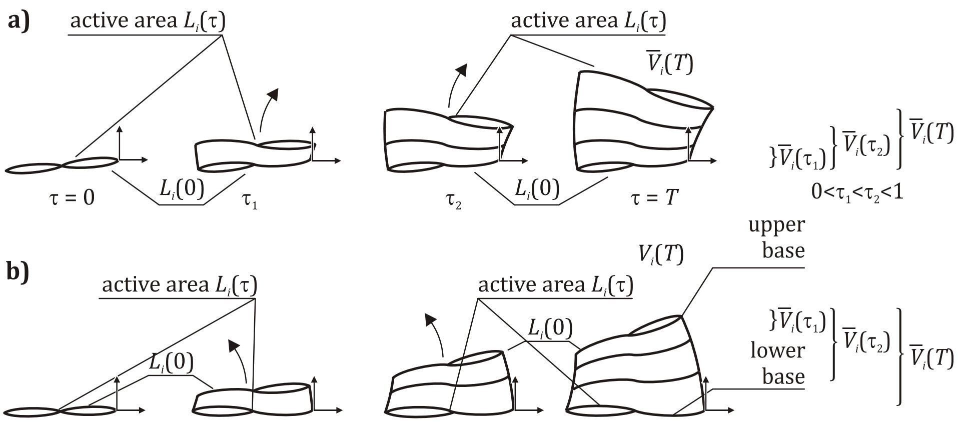

Figure 4: The development of a module a) in the coordinate system of $L_i(0)$ and b) in the coordinate system of the active area, that is $L_i(\tau)$.

At $\tau = 0$ module i is equivalent to its active area $\bar{V}_i(0)$: $\bar{V}_i(0) \equiv L_i(0)$. In the coordinate system of the line $L_i(0)$, we can see the motion of the active area and we can describe it by a motion matrix $\mathbf{M}_i(\tau)$. At the same time, the added layers remain stationary (Fig. 4a). So, the set can be described by:

$$\bar{V}_i(\tau) = \bigcup_{\theta \in [0,\tau]} \mathbf{M}_i(\theta) L_i(\theta).$$

The matrix $\mathbf{M}_i(\theta)$ is called the *growth matrix*. It determines the increase in size of module i as well as its eventual twist or bend. The development of module i can be fully described by a time function for the line $L_i(\tau)$ and a growth matrix $\mathbf{M}_i(\tau)$.

The set $\bar{V}_i(\tau)$ describing the shape of module i in the coordinate system of the line $L_i(0)$ can be transformed to the coordinate system of the active area by a matrix $\mathbf{P}_i(\tau) = \mathbf{M}_i^{-1}(\tau)$. Thus we obtain

$$V_i(\tau) = \mathbf{P}_i(\tau) \bigcup_{\theta = [0,\tau]} \mathbf{M}_i(\theta) L_i(\theta). \tag{2}$$

Fig. 4 shows the growth of module in both coordinate systems. According to the convention used in Fig. 4(b), we call $L_i(0)$ the *upper base* and the active area—*the lower base*.

As mentioned above, for $\tau = 0$ the set $V_i(\tau) = L_i(0)$. On the other hand, for every $\tau > 0$, the set $V_i(\tau)$ can be described as a union of the upper base $L_i(0)$ and the rest $R_i(\tau)$: $L_i(0) \cup R_i(\tau) = V_i(\tau)$. It is clearly seen that the upper base is a part of the module existing whenever a module exists. The upper base changes its position, but does not change its shape. Below, the upper base is denoted by $\bar{L}_i$.

Definition 1 The positioner of the module E_i is a pair $\{\bar{L}_i, \mathbf{P}_i(\tau)\}$. □

The positioner has a constant shape $\bar{L}_i$. In the interval $\tau \in [0, T]$ it changes its position in the coordinate system of the module, which is identified according to the coordinate system of the lower base. For $\tau > T$ the positioner remains stationary.

5. A practical method for creating modules Let us consider the module $V_i(T)$. Bearing in mind that $\mathbf{P}_i(\theta) = \mathbf{M}_i^{-1}(\theta)$ and using Eq. 2, we get

$$V_i(T) = \mathbf{P}_i(T) \left(\bigcup_{\theta \in [0,T)} \mathbf{P}_i^{-1}(\theta) L_i(\theta) \right).$$

The matrix $\mathbf{P}_i(\theta)$ can be decomposed into a translation matrix and a rotation one. The last matrix can be decomposed into three separate matrices describing elementary rotations around the axes x, y and z:

$$\mathbf{P}_i(\theta) = \mathbf{T}_i(\theta)\mathbf{R}_{i,z}(\theta)\mathbf{R}_{i,y}(\theta)\mathbf{R}_{i,x}(\theta).$$

Consider the vertex being the origin of the coordinate system of the upper base. Transform this vertex by the matrix $\mathbf{T}_i(\theta)$ in the interval $\theta \in [0, T]$. In this way, we obtain a line. This line is denoted by the symbol $\{K_i(\theta)\}$ and is called a *path*. For every point of the path denoted by the parameter θ, we know the position of the line $L_i(\theta)$. The matrices $\mathbf{R}_{i,z}(\theta)$, $\mathbf{R}_{i,y}(\theta)$, $\mathbf{R}_{i,x}(\theta)$ describe its orientation. This line describes a tissue fragment formed at the same moment θ as the tissue lying at the point $K_i(\theta)$.

Compound solids created in this way are known in computer graphics as generalized cylinders [4]. Modern graphic tools allow us to create generalized cylinders. Such a cylinder has a finite number of cross-sections, not necessarily planar, distributed along their path in a predefined manner. This tool calculates the surface by interpolating corresponding vertices of subsequent cross-sections to add an edge. This method can be applied to the modules $V_i(T)$, bearing in mind that it does not prevent self-crossing, which cannot happen in natural forms.

The module $V_i(T)$ can be created according to the following algorithm:

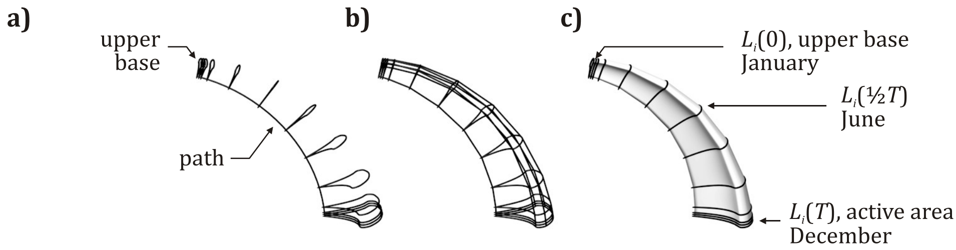

Figure 5: Creating a generalized cylinder for $\theta = T$.

Data: the line $L_i(\theta)$, the positioner matrix $\mathbf{P}_i(\theta)$, a certain sequence of parameters θ_j, where $j = 0, 1, ...J$, $\theta_0 = 0$, $\theta_J = T$ and $\theta_j < \theta_{j+1}$. The set $\{\theta_j\}$ is a sequence of moments in the local time of the module, for which the corresponding cross-sections are taken into account when a generalized cylinder is created.

Algorithm 1

1. Create a line passing through the points $K_i(\theta_j)$; this line is an interpolation of the path $K_i(\theta)$;
2. Create the lines $L_i(\theta_j)$;
3. Create the matrices $\mathbf{P}_i(\theta_j)$;
4. Place every line on the path according to its matrix;
5. Use a generalized cylinder tool in order to create the compound solid.

An example of creating a module is shown in Fig. 5. The shapes of subsequent cross-sections are obtained by transforming the upper base using the following transformations: proportional scaling described by a scale factor $s_{i,j}$, y-axis rotation (matrix $\mathbf{R}_{i,y}(\theta_j)$), z-axis rotation (matrix $\mathbf{R}_{i,z}(\theta_j)$) and shift along an arc-shaped path.

After calculating the shape of the module E_i for $t = T$, we can create this shape for any moment τ. We limit our calculations to the interval $[0, T]$, because for $\tau > T$, the shape is constant, i.e. $V_i(\tau) = V_i(T)$.

As mentioned before, the way a module emerges in nature imposes the property that for any two $\tau_a, \tau_b \in [0, T], \tau_a \neq \tau_b$, cross-sections defined by the lines $L_i(\tau_a)$ and $L_i(\tau_b)$ cannot intersect each other. From this, for any value of the parameter τ ($0 < \tau < T$), the set $V_i(T)$ can be decomposed into two non-intersecting sets $V_i(\tau)$ and $U_i(\tau)$ (i.e. $V_i(\tau) \cup U_i(\tau) = V_i(T)$ and $V_i(\tau) \cap U_i(\tau) = \emptyset$), which can be described as follows:

$$V_i(\tau) = \mathbf{P}_i(\tau)\left(\bigcup_{\theta\in[0,\tau]} \mathbf{P}_i^{-1}(\theta)L_i(\theta)\right), \; U_i(\tau) = \mathbf{P}_i(\tau)\left(\bigcup_{\theta\in[\tau,T]} \mathbf{P}_i^{-1}(\theta)L_i(\theta)\right).$$

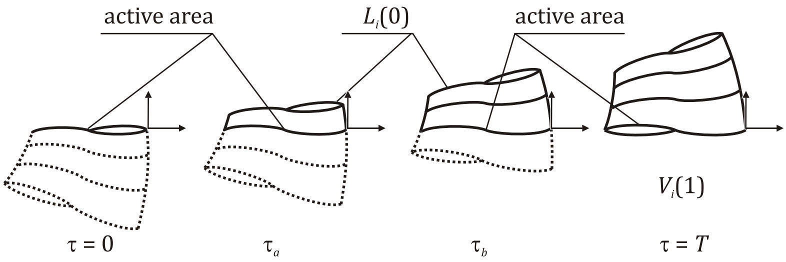

Figure 6: Creating the shape of module E_i by using the set $V_i(T)$ decomposed into two parts: renderable (marked by the solid line) and non-renderable (dashed line).

In computer graphics, instead of the set $V_i(\tau)$, we can use the set $V_i(T)$ divided according to the parameter τ into parts $V_i(\tau)$ and $U_i(\tau)$, where $U_i(\tau)$ is non-renderable (Fig. 6). To achieve this, the set $V_i(T)$ should be equipped with an appropriate animated pixel map in the transparency channel. The boundary between the transparent and non-transparent part of the map should vary according to τ. Often we can use a simpler way, where the part $U_i(\tau)$ of the module is placed inside a skull and due to this is not visible. In this case, the growth process consists only of a progressive sliding out of the complete module from the skull surface. This sliding is described by a positioner matrix.

The situation when the shape of a growing module is a part of the shape of the mature module simplifies the modeling process. This relates to data structures, as well as to manipulating them by an operator.

6. Modularity in the modeling of natural horns Modularity can simplify the modeling process, if copying modules and modifying their parameters is easier than creating modules from scratch. This is the case presented below for creating a ram's horn. Firstly, we construct a parameterized model of the module E_1. Next, we copy modules and at the end we change their parameters.

The parameters of the module and its development are shown in Fig. 7. We have made the following assumptions for the parameters of the module E_1. The module has the shape of a generalized cylinder. The path is an arc of a circle with radius 20 cm and bend angle $\alpha(T) = 75°$. The upper base (the shape of the positioner) is a closed line resembling a triangle. The lower base is at any time a scaled version of the positioner. The movement of the positioner is defined by the angle $\alpha(t) = \alpha(T)f(t)$, where $f(t)$ is a sigmoidal function (see Fig. 7(c)). In the time interval between 0 and T the positioner turns relative to the path of the angle $\beta(t) = \beta(T)f(t)$, where $\beta(T) = 30°$. New modules E_2,... E_8 have been obtained from the module E_1

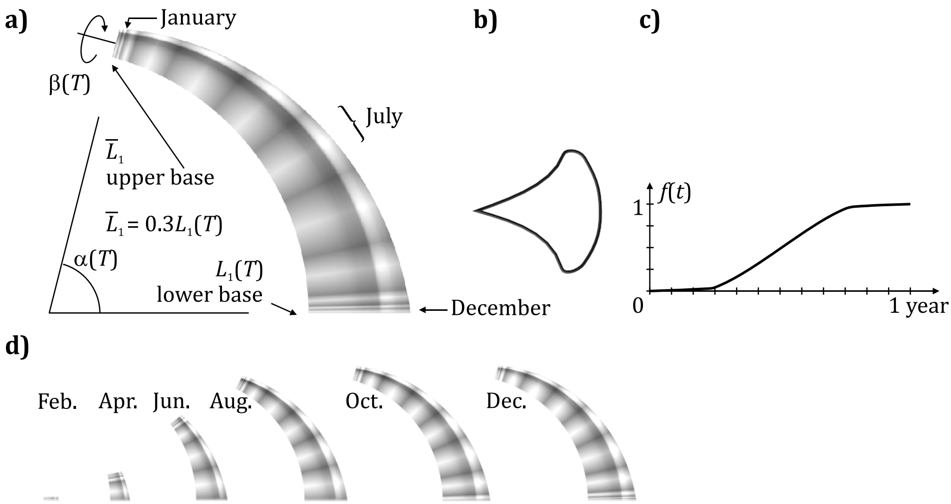

Figure 7: a) the module E_1 and its parameters, b) the shape of the upper base, c) a sigmoidal function and d) the development of the module: monthly increments are marked by stripes—greater in the summer months in the central part of the module.

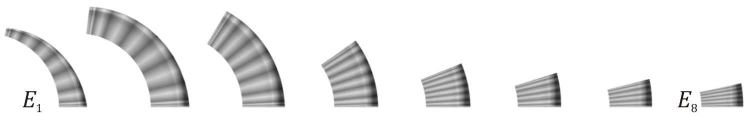

Figure 8: Modules E_1... E_8, each shown at the moment T of its local time.

by changing its parameters according to Table 1. Fig. 8 shows the modules E_1,... E_8, each at the moment T of its local time.

7. Creation of an animated compound solid From the requirement that the lateral surface is continuous, it follows that the upper base of the module E_i is identical to the lower base of the module E_{i-1} both in shape and position, i.e. the positioner of the module E_i describes the motion of the entire structure E_1,... E_{i-1} in the global time interval $t = ((i-1)T, iT]$. The motion of the positioner, that is the matrix $\mathbf{P}_i(\tau)$, can be easily calculated from the matrix $\mathbf{M}_i(\tau)$. This means that even in the case of modules described by more complex time-space functions (defining elongation, bend and twist), we can precisely calculate the position and orientation of a part of the compound solid which emerged earlier.

Let us consider the shape of the compound $Z(t)$, bearing in mind that t, τ and i are related by Eq. 1. The shape of $Z(t)$ can be expressed recursively:

Table 1: Parameters of the modules

i E_i	1 E_1	2 E_2	3 E_3	4 E_4	5 E_5	6 E_6	7 E_7	8 E_8
positioner scale relative to $L_1(T)$ [%]	30	100	137	150	155	156	153	150
scale of $L_i(T)$ relative to $L_1(T)$ [%]	100	137	150	155	156	153	150	148
bend angle $\alpha(T)$ [°]	75	83	59	35	22	17	14	12
twist angle $\beta(T)$ [°]	30	33	24	14	9	8	6	3

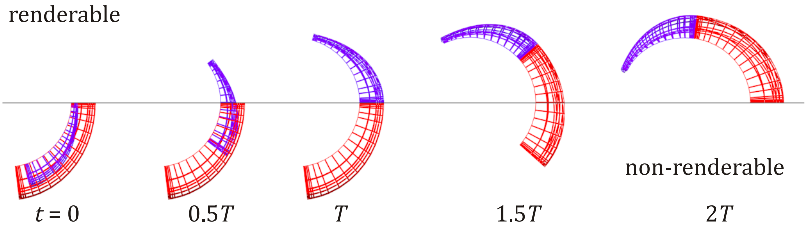

Figure 9: The development of a compound solid in the interval $[0, 2T]$.

$$Z_1(t) = V_1(\tau);$$
$$Z_i(t) = V_i(\tau) \cup \mathbf{P}_i(\tau)\hat{Z}_{i-1} \text{ for } i = 2, 3, \ldots$$

$\hat{Z}_{i-1}$ denotes the shape of the compound solid Z_{i-1} for $t \geq (i-1)T$, that is when the compound solid does not change its shape or position.

Modern graphics tools, eg. 3DS Max [7] [15], allow us to link objects into hierarchical structures and determine the motion of the child object in the coordinate system of its parent. In our case, the motion of the compound solid $Z_1(t)$ is determined by its positioner in the coordinate system of $Z_2(t)$ and consequently, the movement of $Z_2(t)$—in the system of $Z_3(t)$, etc.

In Fig. 9, the development of a compound solid in the interval $[0, 2T]$ is shown. Modules that exist and are renderable are above the horizontal line. Modules (or their parts) that are waiting for their moment to appear, and cannot yet be rendered are below the line.

Figure 10 shows the growth process of a mouflon's horn over eight years. The compound solid consists of 8 modules. Their parameters are placed in Table 1.

8. Discussion In the paper, we have described a method of creating modular models of horns, that uses the concept of a positioner. Modules appear in successive years as an effect of creating new tissue in the active area. The method takes into account the relationship between the rate of tissue formation and seasons. In order to create a compound solid (model),

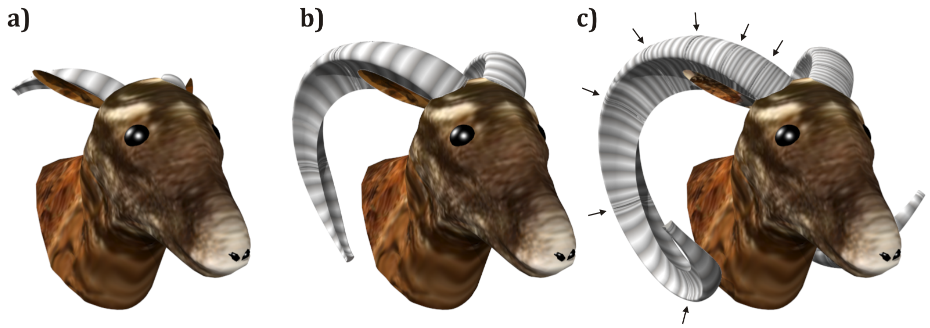

Figure 10: A compound solid a) in the first year, b) after three years and c) after eight years; the arrows point annual rings visible as concentration of stripes.

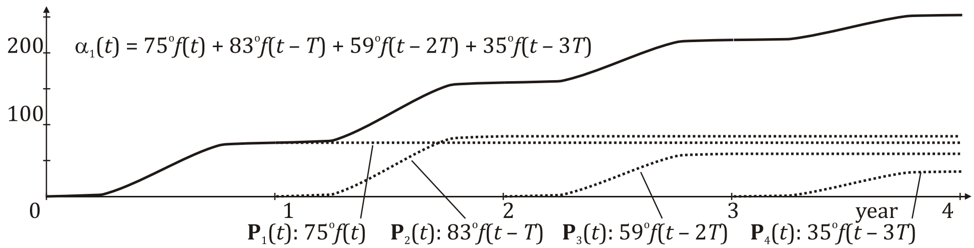

Figure 11: The angle of the positioner i depends on the motions described by the positioners of the modules E_2,... E_4.

it is necessary to create a reference module, which takes into account geometrical and time dependencies. Next, we should copy this module and remake the copies by modifying their parameters. This approach is faster than creating all the modules from scratch. After this, modules must be linked in such a way that a continuous surface and the appropriate motion of the whole compound solid are achieved. This is the point where positioners are useful. They not only describe the motion of the tissue for the currently generated module, but also describe the motion of the previously formed modules. The double role of the positioner allows us to easily create models, even if the geometrical and time dependencies between different parts of the model are complex.

In the example presented in this paper, we assumed that the growth of each module over a year has a sigmoidal character, described by a function $f(t)$. Fig. 11 shows the change of the bending angle of E_1, i.e. the rotation angle around the x-axis in the interval $(0, 4T]$. This rotation is caused by all the positioners existing in the hierarchical structure of the model, i.e. the positioners younger than the considered one. These assumptions imply that at any time only one positioner moves. In order to determine how the bending angle depends on time, we used the function $f(t)$. Thanks to this, we acquired slower growth in winters. This can be observed as a concentration of stripes corresponding to annual rings.

The described method fits the characteristics of bovids' horns better than currently known methods. Also, the modeling process is simple and matched to the capabilities of modern graphics tools.

References

[1] Barnsley M., Jacquin A., Malassenet F., Reuter L., Sloan A. D., *Harnessing Chaos for Image Synthesis*, Computer Graphics (SIGGRAPH '88), 22(4):131–40, doi: 10.1145/54852.378502.

[2] Barnsley M. F., *Fractals everywhere*, 2nd ed. Boston: Academic Press, 1988, ISBN:0-12-079062-9.

[3] Fitzsimmons N.N., Buskirk S.W., Smith M.H., *Population History, Genetic Variability, and Horn Growth in Bighorn Sheep*, Conservation Biology, vol. 9, No 2, April 1995, pp. 314-323, pdf.

[4] Foley J.D., van Dam A., Feiner S. K., Hughes J. F., Phillips R. L., *Introduction to Computer Graphics*, Reading: Addison-Wesley, 1993, ISBN-10: 0201609215.

[5] Lincoln G.A., *Correlation with changes in horns and pelage but not reproduction of seasonal cycles in the secretion of prolactin in rams of wild, feral and domesticated breeds of sheep*, Journal of Reproduction & Fertility. 90(1), 1990: 285-296, doi: 10.1530/jrf.0.0900285.

[6] Lochman J., *Określanie wieku zwierzyny (Determining the age of an animal)*, Państwowe Wydawnictwo Rolnicze i Leśne, Warszawa (in Polish), 1987, ISBN 83-09-00991-7.

[7] Murdock K. L., *3ds Max 2012 Bible*, John Wiley & Sons, ISBN: 978-1-118-02220-7.

[8] Izdebska-Prolejko M., *The modeling of modular plants using positioners on the example of sunflower*, MSc Thesis, Warsaw University of Technology, Warsaw, (in Polish).

[9] Prolejko M., *Using Positional Information in Modeling Inflorescence Discs*, Proceedings of the 8th International Conference on Computer Recognition Systems (CORES 2013), Springer, pp. 71-80, doi: 10.1007/978-3-319-00969-8_7.

[10] Stępień C., *Self-Congruency of Geometric Models of Plants*, Proc. of the VI National Conference on Application of Mathematics in Biology and Medicine. Univ. of Mining and Metallurgy. Zawoja, 12-–15 Sept. 2000, pp. 126-–131, ISBN 83-909553-1-9.

[11] Stępień C., *Self-congruency of models of branched plants*, Proc. of the VII National Conference on Application of Mathematics in Biology and Medicine. Univ. of Mining and Metallurgy. Zawoja, 25-28 Sept. 2001, pp. 161-166, ISBN 83-911926-9-5.

[12] Stępień C., *An IFS-based method for modelling horns, seashells and other natural forms*, Computers & Graphics 33, 576–581. doi: 10.1016/j.cag.2009.02.003.

[13] Stępień C., *Using Positional Information in Modelling Horns, Beaks and Other Natural Forms*, 7th Conference „Computers, Methods and Systems" (CMS'09), Kraków, pp. 457-461, ISBN 83-916420-5-4.

[14] Thompson D. W., *On growth and form*, Cambridge University Press, 1945, pdf.

[15] http://www.autodesk.com/products/3ds-max/overview (retrieved Nov. 12, 2014).

Zastosowanie pozycjonerów do tworzenia modułowych modeli rogów ssaków z rodziny krętorogich

Streszczenie W pracy opisano metodę tworzenia animowanych modułowych modeli rogów ssaków z rodziny krętorogich, w której wykorzystano pojęcie pozycjonerów. W metodzie wykorzystuje się zależne od czasu pozycjonery na dwa sposoby: do ustawiania modułów względem siebie oraz do tworzenia powierzchni bocznej modułu. Dzięki podwójnemu wykorzystaniu pozycjonerów uzyskujemy ciągłość powierzchni bocznej bryły złożonej, bez względu na stopień skomplikowania parametrów zależnych od czasu. Rozważa się różne powiązania parametrów poszczególnych modułów, uzasadnione z punktu widzenia modelowania rogów. Metodę zilustrowano przykładem tworzenia modelu rogu baraniego.

2010 Klasyfikacja tematyczna AMS (2010): Primary 03D20; Secondary 65D18; 68U05.

Słowa kluczowe: grafika komputerowa, animacja, modelowanie, transformacja, modele modułowe, pozycjoner, róg.

Cezary Stępień earned his PhD in technical sciences from the Warsaw University of Technology. His current scientific interests are animated models for computer graphics. He works at Institute of Computer Science, Warsaw University of Technology, where he is a head of postgraduate studies for teachers.

Małgorzata Prolejko graduated from Warsaw University of Technology, The Faculty of Electronics and Information Technology in 2010. Now, she works at The Faculty of Mathematics and Computer Science, University of Warmia and Mazury in Olsztyn. She is interested in 3d computer graphics and researches hierarchical structures and their animated models.

Cezary Stępień
Institute of Computer Science
Warsaw University of Technology
Faculty of Electronics and Information Technology
Nowowiejska str. 15/19, PL-00-665 Warsaw, Poland
E-mail: cst@ii.pw.edu.pl

Małgorzata Prolejko
Faculty of Mathematics and Computer Science
University of Warmia and Mazury
Słoneczna 54 Str., PL-10-710 Olsztyn, Poland
E-mail: m.prolejko@matman.uwm.edu.pl

Communicated by: Mirosław Lachowicz

(Received: 30th of June 2014; revised: 17th of October 2014)