Differential Equations and its Applications
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Integrable Systems and Canonic Quantization

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A number of nonlinear partial differential equations serve as universal physical models, describing a great variety of phenomena. These are soliton-type equations: the Korteweg-de Vries equation, the sine-Gordon equation, the nonlinear Schrödinger equation, the continuum Heisenberg model described by the isotropic Landau-Lifshits equation, the Boussinesq equation, the Toda model etc. They are called completely integrable equations due to infinitely many integrals of motion. Every mentioned equation represents a hierarchy of integrable Hamiltonian systems, and can be constructed on a coadjoint orbit of a loop group by the orbit method. This method gives a powerful apparatus for obtaining periodic and multisoliton solutions for such equations and solving other important problems — one of them is the problem of quantization on a Lagrangian manifold [1], that is a canonical quantization.

The controversial question how to choose a proper Lagrangian manifold for canonical quantization is evidently solved in terms of variables of separation (Darboux coordinates). For integrable Hamiltonian systems constructed by means of the orbit method we have a definite procedure for obtaining variables of separation, they are points of the spectral curve connected to a system. A half of them parametrizes the Liouville torus of the integrable system, thus the Liouville torus serves as a Lagrangian manifold. And the complexified Lagrangian manifold is a generalized Jacobian of the spectral curve, which coincides with the phase space of the system.

The canonical quantization of an integrable system gives a representation of its phase space symmetry algebra over the space of functions on a Lagrangian manifold. Using the Liouville torus as a Lagrangian manifold guarantees that the representation space consists of holomorphic functions — they are defined on the generalized Jacobian serving as the phase space of the system. The obtained representation is indecomposable and non-exponentiated [2].

As an example we consider the nonlinear Schrödinger equation and the continuum Heisenberg model, connecting to the same spectral curve, that allows to use the same Lagrangian manifold for canonical quantization. We construct the corresponding phase space symmetry algebras and their representations over the space of holomorphic functions on the Lagrangian manifold. Harmonic analysis on the representation space leads to the Whittaker equation (in some particular case) and its generalization with an irregular singularity.

References

[1] M. Karasev, *Quantization by membranes and integral representations of wave functions*, Quantization and infinite-dimensional systems. Proceedings of the 12th summer workshop on differential geometric methods in physics, Bialowieza, Poland, 1993, 9–19.

[2] J. Bernatska, P. Holod, *Harmonic analysis on Lagrangian manifolds of Integrable Hamiltonian systems*, Journal of Geometry and Symmetry in Physics, 2013, Vol. 29, 39–51 (arXiv:1307.1785 [math-ph]).