STRUCTURED MODELS FOR MATHEMATICAL EPIDEMIOLOGY

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In recent years we have been working on the formulation an analysis of structured population models for infectious disease dynamics. In contrast to previous models, where for example the age of infection have been used as a structuring variable [6, 8], we introduce structuring of the population with respect to infection (bacterium/virus) load and/or infectiousness.

In this talk we will focus on the models. First we will introduce the so called Wentzell (or Feller) boundary conditions in a structured population model with diffusion. The diffusion in our equation allows us to model random noise in a deterministic fashion. The power of Wentzell boundary condition is that it allows to incorporate a boundary state, which carries mass, namely the population of the uninfected individuals, in an elegant fashion. First we will consider a general linear model [7], then we will consider a nonlinear model which arises when modelling Wolbachia infection dynamics [1]. We establish existence of solutions and consider the existence of positive steady states of the model. If time permits we will briefly mention a general framework we developed recently to treat models with 2-dimensional but non-monotone nonlinearities [2].

In the second part we will introduce and investigate an SIS-type model for the spread of an infectious disease, where the infected population is structured with respect to the different strain of the virus/bacteria they are carrying [3]. Our aim is to capture the interesting scenario when individuals infected with different strains cause secondary (new) infections at different rates, see e.g. [4, 5, 10, 11, 12]. Therefore, we consider a nonlinear infection process, which generalizes the bilinear process arising from the classic mass-action assumption. Our main motivation is to study competition between different strains of a virus/bacteria. From the mathematical point of view, we are interested whether the nonlinear infection process leads to a well-posed model. We use a semilinear formulation (as in [9]) to show global existence and positivity of solutions up to a critical value of the exponent in the nonlinearity. Furthermore, we establish the existence of the endemic steady state for particular classes of nonlinearities.

References

[1] À. Calsina and J. Z. Farkas, Steady states in a structured epidemic model with Wentzell boundary condition, *J. Evol. Equ.*, **12** (2012), 495-512.

[2] À. Calsina and J. Z. Farkas, Postive steady states of nonlinear evolution equations with finite dimensional nonlinearities, *SIAM J. Math. Anal.*, **46** (2014), 1406-1426.

[3] À. Calsina and J. Z. Farkas, On a strain-structured epidemic model., *submitted*.

[4] V. Capasso, Global solution for a diffusive nonlinear deterministic epidemic model, *SIAM J. Appl. Math.*, **35** (1978), 274-284.

[5] V. Capasso and G. Serio, A generalization of the Kermack-McKendrick deterministic epidemic model, *Math. Biosci.*, **42** (1978), 43-61.

[6] A. Ducrot and P. Magal, Travelling wave solutions for an infection-age structured epidemic model with external supplies, *Nonlinearity* **24** (2011), 2891-2911.

[7] J. Z. Farkas and P. Hinow, Physiologically structured populations with diffusion and dynamic boundary conditions, *Math. Biosci. Eng.*, **8** (2011), 503-513.
[8] W. E. Fitzgibbon, M. E. Parrott, and G. F. Webb, A diffusive age-structured SEIRS epidemic model, *Methods Appl. Anal.*, **3** (1996), 358-369.
[9] D. Henry, *Geometric theory of semilinear parabolic equations*, Springer, Berlin-New York, (1981).
[10] A. James, J. W. Pitchford and M. J. Plank, An event-based model of superspreading in epidemics, *Proc. R. Soc. B*, **274** (2007), 741-747.
[11] I. Z. Kiss, J. Cassell, M. Recker, and P. L. Simon, The impact of information transmission on epidemic outbreaks, *Math. Biosci.*, **225** (2010), 1-10.
[12] I. Z. Kiss, P. L. Simon, and R. R. Kao, A contact-network-based formulation of a preferential mixing model, *Bull. Math. Biol.*, **71** (2009), 888-905.

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