Periodic oscillations of a forced pendulum: from existence to stability

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Consider the differential equation

$$x'' + \beta \sin x = f(t)$$

where β is a positive parameter and $f(t)$ is a 2π-periodic function. This is a simple model frequently employed to illustrate the methods of Nonlinear Analysis. Results on the existence of periodic solutions are usually obtained by a combination of tools coming from Topology and Calculus of Variations. The goal of this talk is to show that these tools are also useful in the study of the stability of periodic solutions. Stability is understood in the Lyapunov sense. We will assume that the parameter satisfies $\beta \leq \frac{1}{4}$ and the function $f(t)$ has zero average. The main result says that there exists a stable 2π-periodic solution for almost every periodic function $f(t)$ with zero average. The phrase "for almost every periodic function" is understood in the sense of prevalence. For this reason the notion of set of zero measure in a Banach space of infinite dimensions will play a role.The condition $\beta < \frac{1}{4}$ is sharp. The conclusion of the theorem is not valid "for all periodic functions".