Solving boundary value problems for second order singularly disturbed delay differential equations by ε-approximate fixed-point method

Zbigniew Bartoszewski[1], Anna Baranowska

[1]Gdańsk University of Technology
ul. Narutowicza 11/12, 80-233 Gdańsk, Poland

The presentation will be devoted to numerical solution of boundary value problems for second order singularly perturbed delay differential equations of the form

$$\begin{aligned} \epsilon y''(x) &= f(x, y(x), y'(x), y(\alpha(x))), \qquad a \le x \le b, && (1)\\ y(x) &= \phi(x) \quad \text{for} \quad x \le a, \qquad y(b) = \psi. && (2) \end{aligned}$$

where the functions f, ϕ and α,

$$f : D \to R, \quad D = \{(t, z_1, z_2, z_3) : a \le t \le b, -\infty \le z_i \le +\infty\},$$
$$\phi : [\gamma, a] \to R, \quad \alpha : [a, b] \to (-\infty, b], \quad \gamma = \min_{a \le x \le b} \alpha(x)$$

are continuous and $0 < \epsilon \ll 1$. Problem $(1) - (2)$ is reduced to a linear boundary problem for the system of two equations of the form

$$\begin{aligned} &\mathcal{E}\frac{d\vec{y}(x)}{dx} = g(x, \vec{y}(x), \vec{y}(\alpha(x))), \quad a \le x \le b, && (3)\\ &P_1\vec{y}(x) + P_2(b - a + x)\vec{y}(b - a + x) = \Phi(x), \quad x \in [a - \tau, a], && (4) \end{aligned}$$

with suitably chosen 2×2 matrices P_1, P_2 and the diagonal matrix $\mathcal{E} = \mathrm{diag}\,\{1, \epsilon\}$. For solving problem $(3) - (4)$ we construct a numerical method based on the following theorem [1].

Theorem 1 *If*

a) operator A is continuous in L,

b) the family of spaces S_h and the families of operators r_h and p_h define a convergent approximation of L,

then A possesses at least one fixed-point if and only if there exists a non-negative function $\varepsilon(h)$, $\varepsilon(h) \to 0$ for $h \to 0$ such that the operator A possesses $\varepsilon(h)$-fixed-points $p_h x_h$ and the family $\{p_h x_h \mid h \in M\}$ is compact with $h \to 0$.

Problem $(3) - (4)$ is reduced to a fixed-point with the help of an auxiliary linear problem

$$\begin{aligned} &\vec{y}'(x) = B\vec{y}(x) + \vec{v}(x), \quad x \in [a, b], && (5)\\ &P_1\vec{y}(a) + P_2(b)\vec{y}(b) = \Phi(a), && (6) \end{aligned}$$

and the fixed-point $\vec{v}$ is approximated by a cubic spline. The results of numerical experiments for some examples of problems in the form $(1) - (2)$ will be provided and a comparison with the results obtained with other numerical methods applied to these examples will also be given.

[1] Z. Bartoszewski, *A new approach to numerical solution of fixed-point problems and its application to delay differential equations*, Appl. Math. Comput., 215 (2010), 4320–4331.