Firing map for periodically and almost-periodically driven integrate-and-fire models: a dynamical systems approach

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Abstract

Consider the so-called integrate-and-fire system $\dot{x} = f(t,x)$, $f : \mathbb{R}^2 \to \mathbb{R}$, in which a continuous dynamics govern by the differential equation is interrupted by "the threshold and reset behaviour", meaning that once the value of a dynamical variable reaches a certain threshold it is immediately reset to the resting value and the system evolves again from the new initial condition. The question is to describe the sequence of consecutive resets as iterations of some map, called *firing map* and the sequence of *interspike-intervals* as displacements along a trajectory of this map, which is a notion of importance in various applications, such as e.g. modelling of an action potential (spiking) by a neuron, electric discharges in electrical circuits or cardiac rhythms and arrhythmias.

It turns out that in case of periodic forcing the problem reduces to study of degree-one circle maps. The situation is more complicated when the input is not purely periodic, but almost periodic.